Price of Anarchy in a Two-Sided Critical Distribution System

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Abstract

Measures of allocation optimality differ significantly when distributing standard tradable goods in peaceful times and scarce resources in crises. While realistic markets offer asymptotic efficiency, they may not necessarily guarantee fair allocation desirable when distributing the critical resources. To achieve fairness, mechanisms often rely on a central authority, which may act inefficiently in times of need when swiftness and good organization are crucial. In this work, we study a **hybrid trading system** called *Crisdis*, introduced by Jedličková et al., which combines fair allocation of buying rights with a market – leveraging the best of both worlds. A frustration of a buyer in Crisdis is defined as a difference between the amount of goods they are entitled to according to the assigned buying rights and the amount of goods they are able to acquire by trading. We define a Price of Anarchy (PoA) in this system as a conceptual analogue of the original definition in the context of frustration. Our main contribution is a study of PoA in realistic complex double-sided market mechanisms for Crisdis. The performed empirical analysis suggests that in contrast to market free of governmental interventions, the PoA in our system decreases.

Hybrid Market Structure

We introduce a **new tradable resource called Right**. In each Market, in order to buy the Good, the buyer also needs to possess an equivalent amount of the Right. The Rights are allocated to the buyers before the trading in a Market begins by a centralized *Fairness mechanism* using the sellers' declared offers and buyer's declared Demands. The traders then engage in a series of interactions resulting in their announcement of bids. A dedicated *Market mechanism* then allocates the Goods and Rights based on the bids.

The residual resources of Good after consuming the Demands, and the Money are then transferred to the next Market, as we model the shortage of the critical Good over an extended period of time. The residual of Right disappears after each Market. We call this finite sequence a Crisis.

The **frustration** of buyer b in Market $(\mathcal{M}, \mathcal{G})$ under strategy profile π and allocated rights R is

$$f_b(\pi, \mathcal{M}, \mathcal{G}) = \max\left\{\frac{R_b - G_b}{R_b}, 0\right\}.$$

Price of Anarchy (PoA) in the system is the normalized accumulated frustration the buyers experience in the sequence of $\tau \leq T$ Markets under equilibrium strategy π is

$$PoA^{\tau} = \frac{\sum_{i=1}^{\tau} \sum_{b \in \mathcal{B}} f_b(\pi, \mathcal{M}^i, \mathcal{G}^i)}{\tau |\mathcal{B}|}.$$

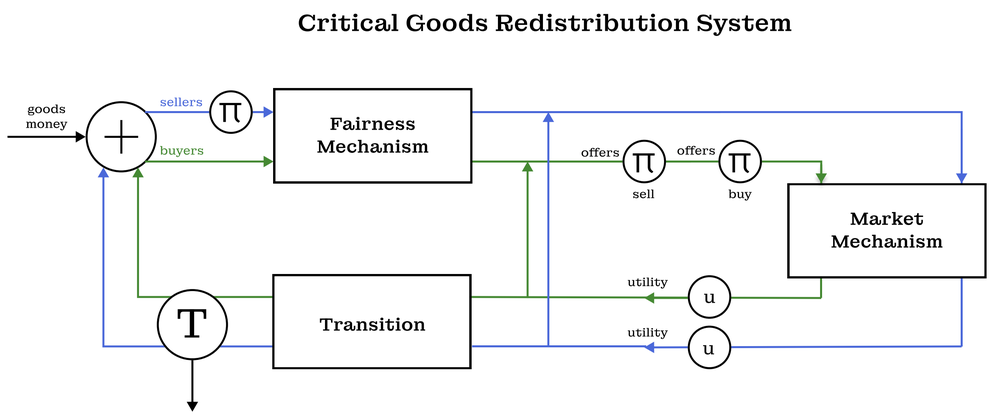


Figure 1. Fairness and market mechanisms positioned in a feedback loop of our redistribution system. One iteration of the outer loop corresponds to one Market. π refers to the strategies and T checks the termination condition.

Results

We demonstrate the properties of our hybrid system with fairness and market mechanisms, and the effectiveness of our learning algorithm, on practical examples. We choose a prototypical setting where three of the four buyers receive significantly more funds then the last buyer. At the same time, this last buyer suffers a large Demand, in most cases exceeding the Demands of the others. We refer to the first three buyers as *rich* and to the last buyer as *poor*.

We measure the quality of a candidate solution from episode τ through its NashConv.

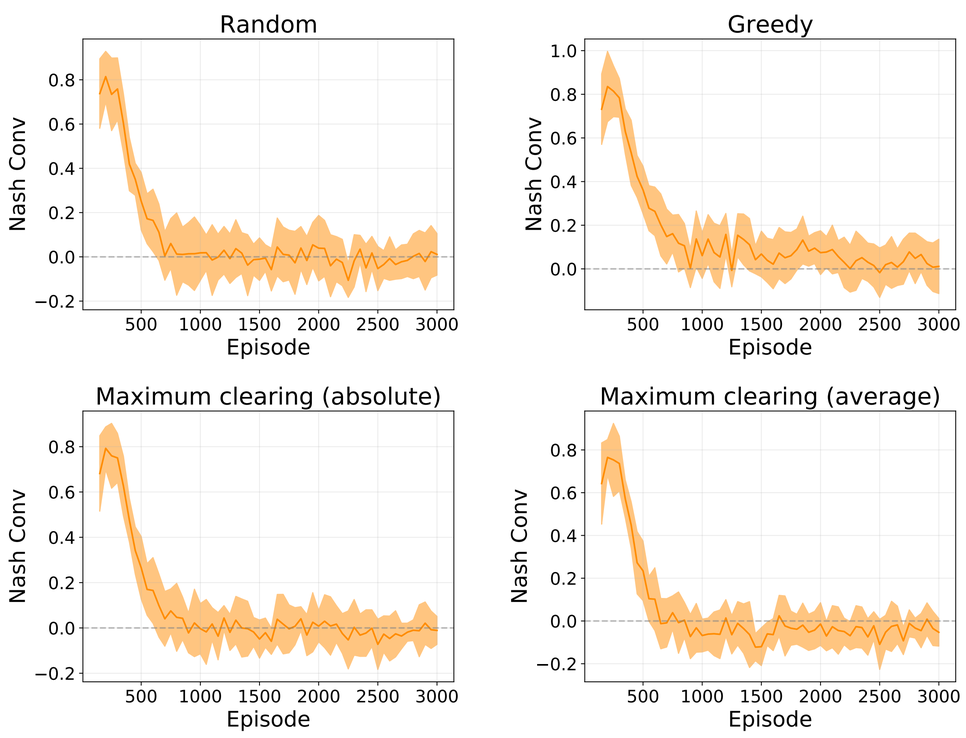
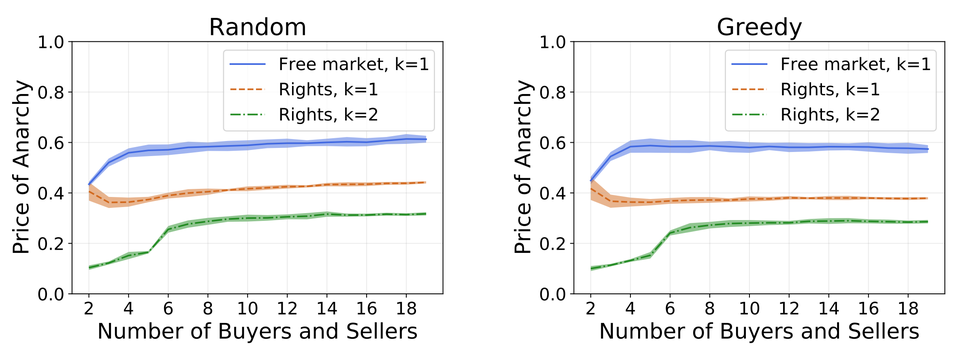


Figure 2. The exploitability of candidate solutions when learning the equilibria in systems with Rights.

Next, we investigate scalability of our hybrid mechanism. The PoA is persistently the largest in the free market. The most interesting cases are the maximum clearing mechanisms where PoA consistently decreases, reaching a value ≈ 6-times smaller than in the free market.



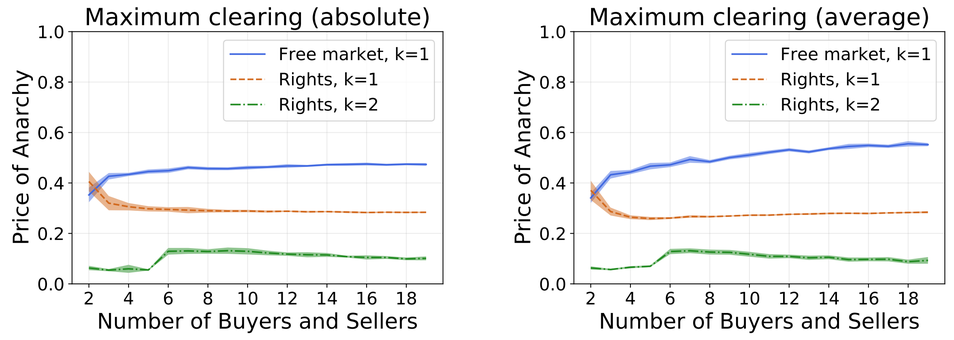


Figure 3. The Price of Anarchy as a function of the number of buyers and sellers.

Finally, we study how the Price of Anarchy evolves in our hybrid system, in comparison to an intervention-free market. Note that the **PoA is always lower in the systems with Rights**. We also show the frustration of each individual buyer.

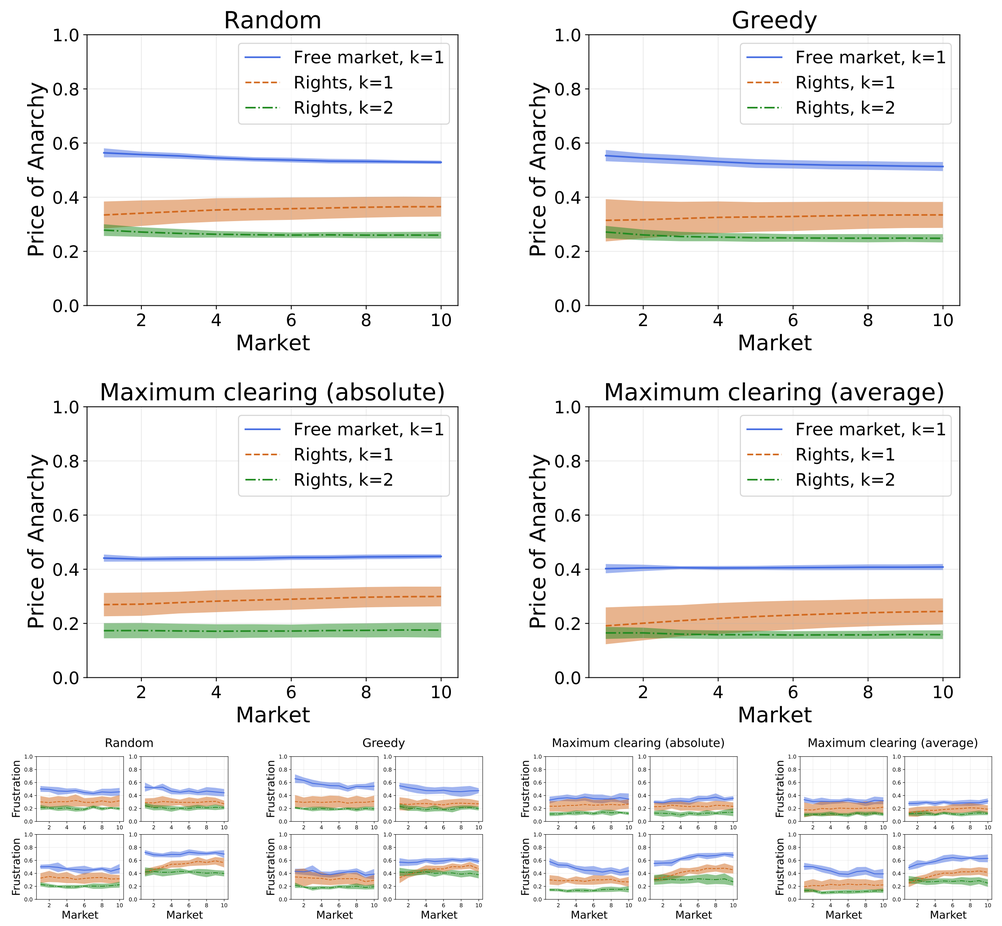


Figure 4. (Top) The Price of Anarchy and (Bottom) the individual frustrations of the four buyers. The poor buyer is always bottom right in the graphs showing individual frustrations.

Conclusion

To the best of our knowledge, we are the first to introduce a system **explicitly combining a double-sided market mechanism with a fairness mechanism** allocating buying rights for more socially just redistribution of critical goods during the times of need. We defined an analogue of Price of Anarchy in our system as a sum of so-called individual frustrations, which are scaled differences between the amount of goods each trader was entitled to according to a fairness mechanism and the amount they were actually able to secure in the market. We show on a notorious example of a system with an underfunded and short-supplied buyer that **introducing the rights may significantly decrease the Price of Anarchy**.

Acknowledgments

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